Novel Derivative of Harmony Search Algorithm for Discrete Design Variables

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Abstract

Calculus has widespread applications in science and engineering. Optimization is one of its major subjects, where a problem can be mathematically formulated and its optimal solution is determined by using derivatives. However, this calculus-based derivative technique can only be applied to real-valued or continuous-valued functions rather than discrete-valued functions while there are many situations where design variables contain not continuous values but discrete values by nature. In order to consider these realistic design situations, this study proposes a novel derivative for discrete design variables based on a harmony search algorithm. Detailed analysis shows how this new stochastic derivative works in the bench-mark function and fluid-transport network design. Hopefully this new derivative, as a fundamental technology, will be utilized in various science and engineering problems.

Keywords: Stochastic Derivative, Harmony Search, Discrete Variable, Combinatorial Optimization, Evolutionary Algorithm, Soft Computing, Metaheuristics

Harmony Search Algorithm

The harmony search algorithm is an optimization technique inspired by music phenomenon. Just as musical instruments are played with certain discrete musical notes based on musicians' experiences or randomness in an improvisation process, so design variables can be assigned with certain discrete values based on computational intelligence or randomness in the optimization process. Just as musicians improve their experiences based on an aesthetic standard, design variables in computer memory can be improved based on objective function.

The original HS algorithm consists of three operations for considering the computational intelligence or randomness as follows (Geem et al., 2001):

$$x_i^{New} \leftarrow \begin{cases} x_i(k) \in \{x_i(1), x_i(2), ..., x_i(K_i)\} & \text{w.p.} \quad P_{Random} \\ x_i(k) \in \{x_i^1, x_i^2, ..., x_i^{HMS}\} & \text{w.p.} \quad P_{Memory} \\ x_i(k \pm m) & \text{w.p.} \quad P_{Pitch} \end{cases} \tag{1}$$

The value of design variable i ($i = 1, 2, \ldots, n$) can be randomly selected from the set of all candidate discrete values $\{x_i(1), x_i(2), ..., x_i(K_i)\}$ with a probability of P_{Random} (random selection); it can be selected from the set of good values $\{x_i^1, x_i^2, ..., x_i^{HMS}\}$ stored in computer memory with a probability of P_{Memory} (memory consideration); or it can be slightly adjusted by moving to neighboring values $x_i(k \pm m)$ once $x_i(k)$ is selected from the set of stored good values, with a probability of P_{Pitch} (pitch adjustment).

Here, the HS algorithm has a memory storage, named harmony memory (HM), where a group of design vectors, $(x_1^j, x_2^j, ..., x_n^j)$, $j = 1, 2, \ldots, HMS$, is stored as many as harmony memory size (HMS). The objective function value is also stored next to each design vector.

$$\mathbf{HM} = \left[\begin{array}{cccc|c} x_1^1 & x_2^1 & \cdots & x_n^1 & f(\mathbf{x}^1) \\ x_1^2 & x_2^2 & \cdots & x_n^2 & f(\mathbf{x}^2) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_n^{HMS} & f(\mathbf{x}^{HMS}) \end{array} \right] \tag{2}$$

The HM is updated with better design vectors with iteration. If newly generated vector $\mathbf{x}^{New}$ is better than the worst vector $\mathbf{x}^{Worst}$ stored in the HM in terms of objective function value, the new vector is swapped with the worst one:

$$\begin{aligned} \mathbf{x}^{Worst} &\notin \mathbf{HM} \\ \mathbf{x}^{New} &\in \mathbf{HM} \end{aligned} \tag{3}$$

This is a basic algorithm structure of HS. Although there are also HS variants for considering the correlation among design variables (Geem, 2006), continuous design variables (Lee and Geem, 2005; Mahdavi et al., 2007), and continuous-valued applications (Li et al., 2006; Ayvaz, 2007; Vasebi, In Press), those variants also follow the above-mentioned three basic features: random selection; memory consideration; and pitch adjustment.

Novel Stochastic Derivative

Based on the HM at certain iteration, the novel partial stochastic derivative of value $x_i(k)$ of variable i can be as follows:

$$\frac{\partial f}{\partial x_i} = \frac{1}{K_i} \cdot P_{Random} + \frac{n(x_i(k))}{HMS} \cdot P_{Memory} + \frac{n\left(x_i(k \mp m)\right)}{HMS} \cdot P_{Pitch} \quad (4)$$

The stochastic derivative for discrete variables gives information that, with how much probability, certain value $x_i(k)$ is selected. The first term in right hand side stands for the probability of random selection; the second term the probability of memory consideration; and the third term the probability of pitch adjustment. The stochastic derivative is the summation of these three terms.

For better understanding, let us consider an objective function $f(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 4)^2$ to be minimized, with the following conditions:

- HMS = 2
- P_{Random} = 0.1
- P_{Memory} = 0.6
- P_{Pitch} = 0.3
- m = 1
- $x_1 \in \{1, 2, 3, 4, 5\}$
- $x_2 \in \{3, 4, 5\}$

The initial randomly-generated HM can be like this:

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 5 & 5 & 10 \end{array}\right] \quad (5)$$

At initial stage, the novel stochastic derivatives for the optimal vector (2, 4) are as follows:

$$\begin{aligned} \left.\frac{\partial f}{\partial x_1}\right|_{x_1=2} &= \frac{1}{5}(0.1) + \frac{0}{2}(0.6) + \left[\frac{1}{2}(0.3)(0.5) + \frac{0}{2}(0.3)(0.5)\right] = 0.095 \\ \left.\frac{\partial f}{\partial x_2}\right|_{x_2=4} &= \frac{1}{3}(0.1) + \frac{0}{2}(0.6) + \left[\frac{1}{2}(0.3)(0.5) + \frac{1}{2}(0.3)(0.5)\right] = 0.183 \end{aligned} \quad (6)$$

For x_1, the optimal value 2 can be selected with a probability of 0.02 in random selection, 0.0 in memory consideration, and 0.075 in pitch adjustment. Here, pitch adjustment is considered by two separate terms: one term for neighboring lower discrete value $x_i(k-m)$ and the other term for neighboring upper discrete value $x_i(k+m)$. The total derivative for x_1 at 2 is 0.095, that is, value 2's probability to be selected at initial stage is 9.5%. In the same fashion, the total derivative for x_2 at 4 is 0.183.

If the HS algorithm selects a new vector, such as (1, 4), at initial stage, this vector can be included in the HM because its function value (1) is better than that (10) of worst vector (5, 5) in the initial HM. The updated HM is as follows:

$$\begin{bmatrix} 1 & 3 & \vdots & 2 \\ 1 & 4 & \vdots & 1 \end{bmatrix} \tag{7}$$

In this updated HM, the stochastic derivatives for the optimal vector (2, 4) are as follows:

$$\begin{aligned} \left.\frac{\partial f}{\partial x_1}\right|_{x_1=2} &= \frac{1}{5}(0.1) + \frac{0}{2}(0.6) + \left[\frac{2}{2}(0.3)(0.5) + \frac{0}{2}(0.3)(0.5)\right] = 0.170 \\ \left.\frac{\partial f}{\partial x_2}\right|_{x_2=4} &= \frac{1}{3}(0.1) + \frac{1}{2}(0.6) + \left[\frac{1}{2}(0.3)(0.5) + \frac{0}{2}(0.3)(0.5)\right] = 0.408 \end{aligned} \tag{8}$$

As observed, the chances to select optimal values normally increase with iteration. Thus, the HS algorithm can ultimately find the optimal solution or near-optimal solutions by the help of this stochastic derivative for discrete variables.

Bench-Mark Example

In order to observe the performance of the stochastic derivative, let us consider the following minimization function with one discrete variable:

$$f(x_1) = (x_1 - 3.1)^2 \tag{9}$$

The conditions for HS computation are as follows:

- HMS = 3
- P_{Random} = 0.30

- $P_{Memory} = 0.49$
- $P_{Pitch} = 0.21$
- $m = 1$
- $x_1 \in \{1, 2, \ldots, 10\}$

The HS computation started with the initial HM of $[8, 6, 6]^T$, evolved as $[1, 6, 6]^T$ at 3rd iteration, $[1, 1, 2]^T$ at 9th iteration, $[4, 2, 2]^T$ at 20th iteration, $[4, 3, 2]^T$ at 25th iteration, $[3, 3, 3]^T$ at 28th iteration.

Figure 1 shows the changes of stochastic derivative values at different iterations. While the value 6 and neighboring values initially had higher chances to be selected, the optimal value 3 and neighboring values eventually has higher chances with increasing iteration.

Figure 2 shows the stochastic derivative values of three components (random selection, memory consideration, and pitch adjustment) especially at 25th iteration which has the HM of $[4, 3, 2]^T$.

The HS algorithm was able to find the optimal solution (3) after 28 trials (three of initial random generations and 25 of harmony generations). Although the number (28) of this trials is more than that (10) of total enumeration, this number is drastically reduced when the algorithm is applied to huge combinatorial problems. When a variable has 100 discrete values, the algorithm found the optimal solution after average 101.8 iterations (standard deviation = 135.0); when a variable has 1000 discrete values, the algorithm found the optimal solution after average 333.3 iterations (standard deviation = 193.8).

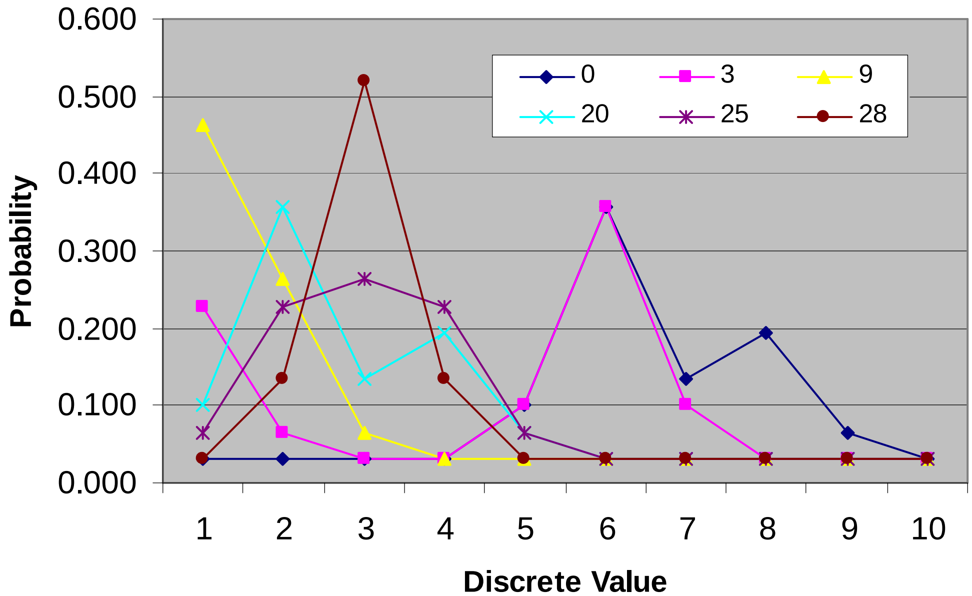

Figure 1. Changes of Stochastic Derivative

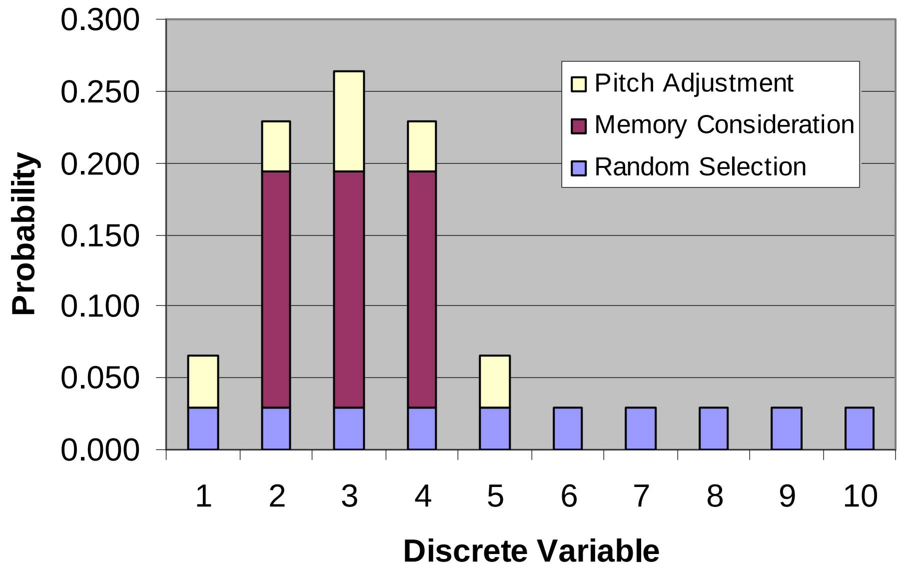

Figure 2. Stochastic Derivative at 25th Iteration

Real-World Example

The stochastic derivatives are also observed in real-world engineering problems such as water network design (Geem, 2006a), as shown in Figure 3.

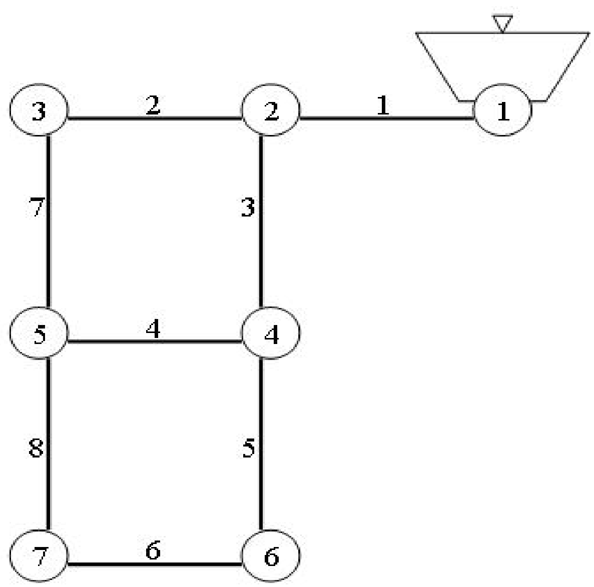

Figure 3. Schematic of Two-Loop Water Distribution Network

The network contains seven nodes, eight pipes, and two loops. While demand nodes (node 2 to node 7) require adequate water amounts and water pressures, supply node (node 1) should satisfy the water amount, and pipe diameters should be large enough to satisfy the water pressures. The goal of the problem is to find minimal diameter of each pipe while satisfying water amount and pressure constraints. The objective function is as follows:

$$\text{Minimize } z = \sum_{i=1}^{n} f(D_i, L_i) \tag{10}$$

where D_i and L_i are respectively diameter and length for pipe i .; $f(\cdot)$ is a cost function for given diameter and length; and n is the number of pipes in a network.

For the two-loop network problem, each pipe has a set of 14 candidate diameters {1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24} (Diameter unit is inch). Because the number of pipes in the network is eight, the number of possible designs is 1.48×10^9 (= 14^8). The HS algorithm, however, found the global optimal solution (18, 10, 16, 4, 16, 10, 10, 1) with the cost of $419,000 after only 1,234 iterations, by the help of stochastic derivatives.

Table 1. Stochastic Derivative Values in Pipe 1

Diameter (Inch)	Iterations					
	0	26	47	141	585	1,234
1	0.0971	0.0071	0.0071	0.0071	0.0071	0.0071
2	0.1061	0.0071	0.0071	0.0071	0.0071	0.0071
3	0.1601	0.0071	0.0071	0.0071	0.0071	0.0071
4	0.0341	0.0071	0.0071	0.0071	0.0071	0.0071
6	0.0881	0.0206	0.0071	0.0071	0.0071	0.0071
8	0.0926	0.1421	0.0251	0.0071	0.0071	0.0071
10	0.0611	0.2501	0.1646	0.0071	0.0071	0.0071
12	0.0836	0.1511	0.1466	0.0071	0.0071	0.0071
14	0.0206	0.1016	0.1511	0.0746	0.0071	0.0071
16	0.0476	0.0926	0.2096	0.5651	0.0836	0.0881
18	0.0476	0.0566	0.1061	0.2231	0.6326	0.6641
20	0.0161	0.0476	0.0521	0.0611	0.1916	0.1601
22	0.0521	0.0206	0.0206	0.0116	0.0206	0.0161
24	0.0926	0.0881	0.0881	0.0071	0.0071	0.0071
Σ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 1 and Figure 4 (a) shows the changes of stochastic derivatives in pipe 1. For pipe 1, 3-inch diameter initially has highest chance (0.1601) to be selected. At 26th iteration, 10-inch diameter has highest chance (0.2501); at 47th and 141st iterations, 16-inch diameter has highest chance (0.2096 and 0.5651); and 585th and 1,234th iterations, 18-inch diameter, which is the optimal diameter for the pipe, finally has highest chance (0.6326 and 0.6641). For other pipes, the situation is similar. Either the optimal diameter had highest chance or it had one of high chances.

The probability that the HS algorithm selects the optimal solution based on current HM is as follows:

$$\prod_{i=1}^{n} \left. \frac{\partial f}{\partial x_i} \right|_{x_i = x_i^*} \tag{11}$$

where x_i^* is an optimal value for design variable i. Using Equation 11, the probability to select the optimal solution vector based on initial HM is 7.47 × 10^{-10}. However, the probability is drastically increased up to 0.0049 based on the HM of 1,234th iteration. That is, the algorithm has a chance to find the optimal solution vector every 200 iterations.

Conclusions

This study proposes a new stochastic derivative for discrete variables based on a harmony search algorithm. While traditional calculus-based derivative gives information of search direction and step size at certain single vector for a function which has continuous variables (Mays and Tung, 1992), the stochastic derivative in this study gives information of probabilistic inclination to select certain discrete point based on multiple vectors stored in HM for a function which has discrete variables.

Also, detailed analysis is performed to show how this new stochastic derivative works in a bench-mark example and real-world problem. This stochastic derivative became a good tool to search the optimal solution. With iteration, optimal and neighboring values had higher chances to be selected. For the fluid-transport network design, the HS algorithm was able to find the global optimum solution after only 1,234 function evaluations out of total 1.48 × 109 combinations by the help of the stochastic derivative.

In addition to the fact that the stochastic derivative is useful for discrete variables, it is also useful when function's mathematical derivative cannot be analytically obtained or when function's type is step-wise or condition-wise. Thus, this stochastic derivative is expected to be applied to even more various scientific and engineering problems.

For future research, the stochastic derivative information should be utilized more efficiently and effectively. Also, in order to consider relationship among variables, stochastic co-derivative may be devised.

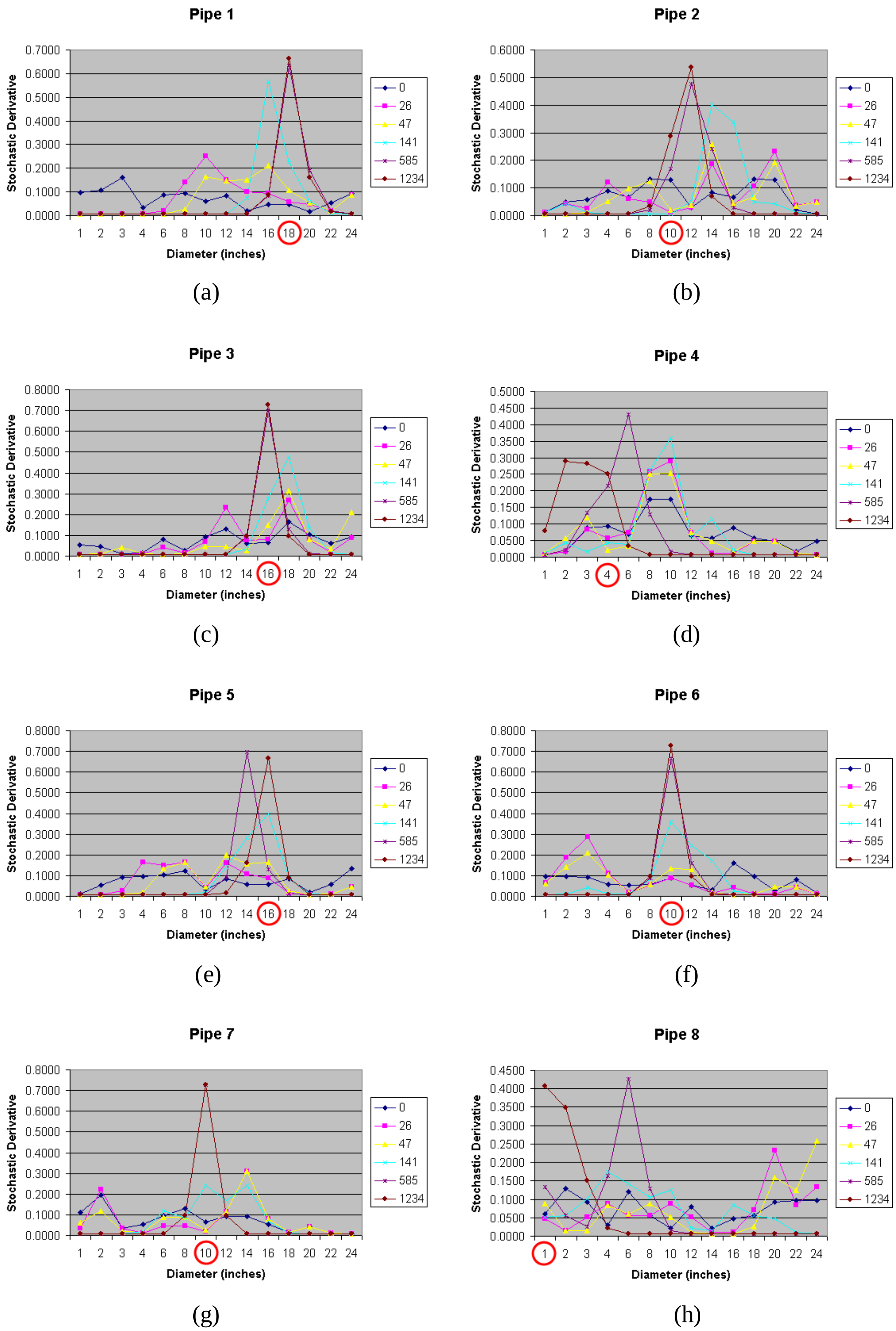

Figure 4. Changes of Stochastic Derivatives in Each Pipe

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